

Hanbury Brown and Twiss correlations in atoms scattered from colliding condensates

Klaus Mølmer

*Lundbeck Foundation Theoretical Center for Quantum System Research,
Department of Physics and Astronomy, University of Aarhus, DK-8000 Århus C, Denmark*

A. Perrin, V. Krachmalnicoff, V. Leung, D. Boiron, A. Aspect, C.I. Westbrook

*Laboratoire Charles Fabry de l'Institut d'Optique, CNRS, Univ Paris-sud
Campus Polytechnique, RD128, 91127 Palaiseau, France*

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Low energy elastic scattering between clouds of Bose condensed atoms leads to the well known s-wave halo with atoms emerging in all directions from the collision zone. In this paper we discuss the emergence of Hanbury Brown and Twiss coincidences between atoms scattered in nearly parallel directions. We develop a simple model that explains the observations in terms of an interference involving two pairs of atoms each associated with the elementary s wave scattering process.

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I. INTRODUCTION

In a number of experiments, Bose-Einstein condensates have been prepared to collide with each other with well defined collision energies and momenta. At the microscopic level, when two particles with equal mass and opposite velocities collide in an s-wave collision, the collision partners will propagate away from each other with the same probability amplitude in all directions, but their individual momenta are correlated in opposite directions, as their total center-of-mass momentum is conserved in the collision process. In experiments with colliding condensates, the scattering into all directions has been clearly observed as a so-called s-wave halo of scattered particles[1]. The observation of pair correlations of particle leaving the collision region back-to-back, see Fig. 1, requires efficient detection of all momentum components of individual atoms, and this correlation has recently been observed as a significant coincidence signal in a collision experiment with Bose-Einstein condensates of metastable atomic helium [2]. The same experiment also observed an increased coincidence of particles scattered into *the same* direction. This phenomenon is due to the bosonic nature of the particles and to the fact that several independent scattering processes occur simultaneously.

We shall present a theoretical analysis of this Hanbury Brown and Twiss correlation phenomenon, aiming at a simple model which explains its qualitative and quantitative character in experiments. It is important to emphasize that the appearance of atoms moving in the same direction is not compatible with momentum conservation in a single collision process of two counterpropagating atoms, and our discussion will, indeed, refer to effects that rely on a many-body treatment of the collision of larger ensembles. In order to get physical insight, we will separate the problem in two: first, we treat independent pairwise collisions in which the collisional interaction gives rise to pairs of scattered atoms, and second, we treat the evolution of the many-body state describing the ensemble of scattered atoms neglecting the interactions

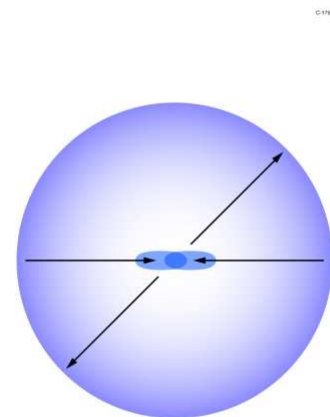


FIG. 1: Diagrammatic representation of two condensates colliding and giving rise to an s-wave halo of scattered particles. Particles are scattered pairwise back-to-back.

in order to apply analytic methods. The validity of this separation and means to improve the theory, if necessary, will be discussed.

In Sec. II, we present a full second quantized description of the collision of a large number of identical bosons. We shall write down the second quantized many-body Hamiltonian, and discuss how the elementary processes of interest relate to the different terms in this Hamiltonian. We treat the case where all the atoms initially populate two counter propagating single particle states which are only weakly depleted by the collisions, and which will hence serve as c-number field sources for creation of pairs. This is in analogy with the quantum optics

treatment of spontaneous four-wave mixing, where photon pairs are generated from the interaction of two incident laser beams, described by classical electromagnetic waves. In Sec. III, we discuss the Bogoliubov transformation which provides a very accurate approximation to the time evolution of the system. We shall not, however, apply this transformation in a quantitative treatment, but rather show that its formal structure already predicts the collinear (Hanbury Brown and Twiss) correlation and motivates a quite general analytical Ansatz for the quantum state of the scattered atoms. In Sec. IV, we shall consider the leading two- and four-atom terms in an expansion of the quantum state of scattered atoms, and show that they hold the key to the observed Hanbury Brown and Twiss correlations. In Sec. V, we shall use energy conservation and phase matching considerations to motivate a simple analytical model, from which we show that the coincidence of scattered particles in the same direction, though a many-body effect, can be understood quantitatively from the properties of the simple two-atom scattering wave function. Sec. VI concludes the paper with a discussion of the insights offered by our analysis.

II. COLLIDING BOSE-EINSTEIN CONDENSATES

The Hamiltonian,

$$H = \int d\vec{r} \hat{\Psi}^\dagger(\vec{r}) \left(-\frac{\hbar^2}{2m} \Delta + V(\vec{r}) + \frac{g}{2} \hat{\Psi}^\dagger(\vec{r}) \hat{\Psi}(\vec{r}) \right) \hat{\Psi}(\vec{r}), \quad (1)$$

with field operators obeying the bosonic commutator relations $[\Psi(\vec{r}), \Psi^\dagger(\vec{r}')] = \delta(\vec{r} - \vec{r}')$, gives a good description of bosons interacting at low collision energies via a short range potential. The interaction is represented by a delta-function interaction term with strength g , proportional to the s-wave scattering length. The atoms may be subject to a wide range of trapping or guiding potentials $V(\vec{r})$, or they may propagate freely ($V = 0$), and the initial state of the system may be specified according to experimental preparation procedures to describe for example a single condensate or several macroscopically populated components. We are interested in the situation, where two condensates with well defined opposite momenta, and hence with relatively large spatial extent propagate towards each other. The conventional second quantized Hamiltonian fully describes the problem, and a Monte-Carlo type simulation of the dynamics [3, 4, 5], and perhaps even simpler simulation approaches based on truncated Wigner function expansions [6, 7], may solve this problem in full generality by full 3D propagation of stochastic Schrödinger type equations.

We assume that elastic collisions occur with a sufficiently small cross section that the colliding condensates are only weakly depleted due to the collision term in (1). The Hamiltonian has terms describing the kinetic energy

and the potential energy of atoms moving in the external potential and finally, a term describing the mean field repulsive or attractive potential due to the other atoms of the colliding condensates. But the product of two creation and two annihilation operators in the interaction term does not only read as density dependent correction to the potential energy in the Gross-Pitaevskii equation: the product of two creation operators may also cause the creation of a pair of atoms with momenta entirely different from the incident ones, extracted consistently from the condensates by the product of annihilation operators. The pairs of atoms “created” in the scattering process are the ones that are detected as the s-wave halo around the condensate collision region in Fig. 1.

We can think of each point in the collision zone as a point source for a pair of initially close atoms (atoms only collide at short range), which are subsequently separated by free propagation, perturbed by the interaction with the condensate components. This propagation, together with the coherent addition of pair amplitudes originating from the entire collision zone leads to a complicated many-body entangled state, but energy conservation, imposed after sufficiently long interaction time, and momentum conservation, imposed by phase matching, serves to justify our simpler model, described below.

III. BOGOLIUBOV APPROXIMATION

If the original condensates are only weakly depleted by the scattering, we may expand the Hamiltonian to second order in operator terms and get linear Heisenberg equations of motion that may be solved and tell precisely which Bogoliubov transformation exactly solves the problem. This approach was followed in Refs. [8, 9, 10]. The Bogoliubov solution yields an expression for the atomic annihilation operators at any given time, expressed as a linear combination of the annihilation and creation operators at time zero, where the initial state is assumed to be known (incident condensate wave functions, no scattered atoms). The mean atom number and any higher order correlation function of the field can therefore be expressed algebraically in terms of the expansion coefficients of the Bogoliubov transformation and the known vacuum expectation values of field operator products.

Even though the full many problem has thus been reduced to partial differential equations of a complexity comparable to the single particle Schrödinger equation, one still has to solve time dependent wave equations in three spatial dimensions. Here we shall demonstrate some properties of the solution that follow by purely analytical arguments, i.e., without access to the precise solution.

Although obviously related, the use of the Bogoliubov transformation here is different from the Bogoliubov approximation used to identify low-lying, collective excitation modes in a condensate. The analysis rather follows the philosophy of squeezed light generation with opti-

cal parametric oscillators in quantum optics, where the Bogoliubov method is used to diagonalize a multi-mode Hamiltonian with pair creation and annihilation operators [11], see also [14, 15].

Since the initial state of the atomic scattering modes (momentum components) of interest is the vacuum state, which has a Gaussian (Wigner) probability distribution for the multi-mode field variables, and the Bogoliubov transformation is linear in field operators, the state will, independently of the precise form of the transformation, at all later times be a Gaussian with vanishing mean field expectation value [16]. If we restrict the analysis to a single final momentum state (mode), by a partial trace over all other modes, the state of this mode is also a Gaussian state with vanishing mean amplitude. It is thus fully characterized by the second moments of the hermitian linear combinations $q_{\vec{k}} \equiv (\hat{\Psi}^\dagger(\vec{k}) + \hat{\Psi}(\vec{k}))/\sqrt{2}$, $p_{\vec{k}} \equiv i(\hat{\Psi}^\dagger(\vec{k}) - \hat{\Psi}(\vec{k}))/\sqrt{2}$ of the field operators. We now wish to establish that our Gaussian distribution is symmetric, *i.e.*, $\text{Var}(q_{\vec{k}}) = \text{Var}(p_{\vec{k}})$. This indeed follows if the "anomalous" moments $\langle \hat{\Psi}^\dagger(\vec{k})^2 \rangle = \langle \hat{\Psi}(\vec{k})^2 \rangle = 0$, *i.e.*, if there is no coherence between states differing by two atoms propagating in the given direction. We now apply the physical argument, that the collisional Hamiltonian does not produce such coherence, since the collision process can only produce pairs of atoms propagating in opposite directions, and states, *e.g.*, with zero and two atoms with momentum \vec{k} must also contain zero and two atoms with momentum around $-\vec{k}$. The anomalous moments vanish due to the orthogonality of these parts of the wave function.

It is well known in quantum optics, that a symmetric Gaussian state is equivalent to an incoherent mixture of number states with exponential number distribution, also known as a thermal state with the density matrix [11],

$$\rho_1 = (1 - |t|^2) \sum |t|^2 |n\rangle \langle n|. \quad (2)$$

The state conditioned upon detection, and annihilation, of a single particle reads,

$$\rho_c = \nu \hat{a} \rho_1 \hat{a}^\dagger = \frac{(1 - |t|^2)^2}{|t|^2} \sum_n |t|^2 |n\rangle \langle n-1| \langle n-1|, \quad (3)$$

where ν is a normalization constant. A straightforward calculation shows that this state has precisely twice as many bosons on average as (2), and hence that the probability to detect two bosons by a low efficiency detector is twice the square of the single quantum detection probability. It thus follows that the coincidence counting rate for observing two atoms leaving the collision zone in the same, narrowly defined, direction, $\langle \hat{\Psi}^\dagger(\vec{k}) \hat{\Psi}^\dagger(\vec{k}) \hat{\Psi}(\vec{k}) \hat{\Psi}(\vec{k}) \rangle$ is twice the square of the mean counting rate, and twice the coincidence rate for seeing atoms in two unrelated directions.

Without performing any calculations, we therefore understand qualitatively the observed coincidences observed in the experiments [2] as the direct consequence

of the thermal counting statistics (Gaussian quadrature distribution) of the output flux in all scattering directions. This is the famous Hanbury Brown and Twiss effect [17, 18, 19, 20] observed originally as photon bunching in chaotic light resulting from the addition of the contributions of many incoherent emitters. In order to provide a natural estimate of the HBT momentum correlation function, one could develop the field correlations by solution of the linear Bogoliubov-de-Gennes equations for the problem [8, 9, 10] which by the corresponding linear transformation of operators provides the first and second order momentum correlation functions and hence the momentum range within which the bunching effect takes place. Here, we will rather keep track of the binary scattering states, and in particular of the counterpropagating partners, which will give us an alternative and very useful physical interpretation of the effect.

The Bogoliubov transformation of field operators is equivalent to a multi-mode unitary squeezing operation [11], which is indeed nothing but the time evolution operator of a Hamiltonian with quadratic terms in field creation and annihilation operators. Such an operator can be ordered as a product of three exponentials [12, 13]: one involving a sum of products of pairs of creation operators, one involving a sum of products of creation and annihilation operators and one involving a sum of products of pairs of annihilation operators. When acting on the initial vacuum state vector, only the unit term of the series expansion of the latter two exponentials contribute, and the state can therefore be written in terms of a quadratic form of creation operators of atoms, *e.g.*, in the momentum space representation,

$$|\Psi\rangle = N_\Psi \exp\left(\int d\vec{k}_1 d\vec{k}_2 \psi(\vec{k}_1, \vec{k}_2) \hat{\Psi}^\dagger(\vec{k}_1) \hat{\Psi}^\dagger(\vec{k}_2)\right) |vac\rangle. \quad (4)$$

The function $\psi(\vec{k}_1, \vec{k}_2)$ generally depends in a complex manner on the dynamical evolution. It is of course related to the scattering wave function of a single pair of atoms, and we shall come back to this relationship in connection with the model studied in Sec. V of the paper. The use of second quantization automatically yields the bosonic symmetry of our state, but in addition we can require that the pair amplitude function obey the explicit exchange symmetry $\psi(\vec{k}_1, \vec{k}_2) = \psi(\vec{k}_2, \vec{k}_1)$. For now, let us assume, that the propensity for atoms to be scattered into opposite direction also implies that $\psi(\vec{k}_1, \vec{k}_2)$ takes non-vanishing values for all directions of the scattered particles, but only if $\vec{k}_1 \sim -\vec{k}_2$. The function $\psi(\vec{k}_1, \vec{k}_2)$ is not a normalized wave function: the larger its amplitude the more particle pairs are created, and higher order terms of the exponential play more and more important roles. The many-body state $|\Psi\rangle$ is normalized by the prefactor N_Ψ in (4).

We now proceed to determine the density-density correlations of atoms detected in two different directions, labeled by momentum states (\vec{k}, \vec{k}') , *i.e.*, the expectation

value

$$F(\vec{k}, \vec{k}') \propto \langle \hat{\Psi}^+(\vec{k}) \hat{\Psi}^+(\vec{k}') \hat{\Psi}(\vec{k}') \hat{\Psi}(\vec{k}) \rangle \quad (5)$$

The state (4) is a Gaussian state, and by Wick's theorem [21] this expectation value can be written down in terms of only pair-expectation values. We shall address the contribution from the four-atom component in the expansion of the exponential in (4), as this provides a straightforward interpretation of the origin and the behavior of the atomic Hanbury Brown and Twiss correlations.

IV. TWO-ATOM AND FOUR-ATOM STATES

The state (4) can be written explicitly,

$$\begin{aligned} |\Psi\rangle &= N_\Psi (|vac\rangle \\ &+ \int d\vec{k}_1 d\vec{k}_2 \psi(\vec{k}_1, \vec{k}_2) \hat{\Psi}^+(\vec{k}_1) \hat{\Psi}^+(\vec{k}_2) |vac\rangle \\ &+ \frac{1}{2} \left(\int d\vec{k}_1 d\vec{k}_2 \psi(\vec{k}_1, \vec{k}_2) \hat{\Psi}^+(\vec{k}_1) \hat{\Psi}^+(\vec{k}_2) \right)^2 |vac\rangle \\ &+ \dots). \end{aligned} \quad (6)$$

The zero order term is the vacuum state. The first order term is a two-atom state of atoms propagating back-to-back, and the second order term of the series expansion of (4) is the four-atom state

$$|\Psi_4\rangle \equiv \left(\int d\vec{k}_1 d\vec{k}_2 \psi(\vec{k}_1, \vec{k}_2) \hat{\Psi}^+(\vec{k}_1) \hat{\Psi}^+(\vec{k}_2) \right)^2 |vac\rangle, \quad (7)$$

which we will show accounts for the observed HBT effect. The squared pair creation operator in (7) can be expanded as a four-fold integral. To obtain the correlation function (5), we have to apply the product of the two annihilation operators on $|\Psi_4\rangle$ and determine the squared norm of the resulting state,

$$F(\vec{k}, \vec{k}') \propto \|\hat{\Psi}(\vec{k}) \hat{\Psi}(\vec{k}') |\Psi_4\rangle\|^2. \quad (8)$$

Using the field commutator relations, we can shift the annihilation operators to the right of all creation operators in (8). This yields a total of 12 terms, which by relabeling and use of the exchange symmetry can be reduced to a sum of three different contributions,

$$\begin{aligned} &\hat{\Psi}(\vec{k}) \hat{\Psi}(\vec{k}') |\Psi_4\rangle \propto \\ &\int d\vec{k}_1 d\vec{k}_2 \{ \psi(\vec{k}_1, \vec{k}_2) \psi(\vec{k}, \vec{k}') + \psi(\vec{k}_1, \vec{k}) \psi(\vec{k}_2, \vec{k}') \\ &+ \psi(\vec{k}_1, \vec{k}') \psi(\vec{k}_2, \vec{k}) \} \hat{\Psi}^+(\vec{k}_1) \hat{\Psi}^+(\vec{k}_2) |vac\rangle. \end{aligned} \quad (9)$$

and thus its squared norm:

$$\begin{aligned} F(\vec{k}, \vec{k}') &\propto \int d\vec{k}_1 d\vec{k}_2 | \psi(\vec{k}_1, \vec{k}_2) \psi(\vec{k}, \vec{k}') \\ &+ \psi(\vec{k}_1, \vec{k}) \psi(\vec{k}_2, \vec{k}') + \psi(\vec{k}_1, \vec{k}') \psi(\vec{k}_2, \vec{k}) |^2 \end{aligned} \quad (10)$$

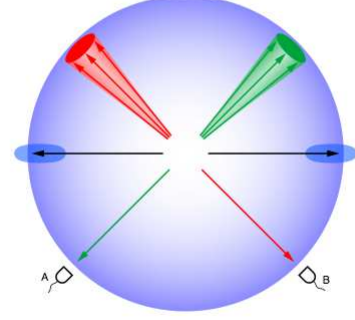


FIG. 2: State of four atoms, scattered pairwise back-to-back. Atoms detected by detectors A and B in an arbitrary pair of directions have partners recoiling in the opposite directions within a certain width imposed by the uncertainty on total and relative momentum of the atoms. The quantum state of the detected pair is obtained by a partial trace over the recoiling momentum components, and the coincidence counting yield in the detectors is just the product of the single detector count signals

This is the main result of the paper. Dealing explicitly with the four atom component it is easy to see what happens. In Fig. 2. we illustrate the case of detection of a particle pair in random directions. Because, as noted below Eq. (4), $\psi(\vec{k}, \vec{k}')$ vanishes unless \vec{k} and \vec{k}' are anti-parallel, the first term in Eq. (10) only contributes if the detectors correspond to opposite directions. For opposite or random directions such as in Fig. 2, there is also no cross term between the second two terms because one vanishes whenever the other is finite. If (\vec{k}, \vec{k}') are nearly parallel, as illustrated in Fig. 3, the last two terms evaluate the two different ψ -terms at the detector directions and at the direction specified by the integration variables. This means that values of the integration variables \vec{k}_1, \vec{k}_2 exist (opposite to the detector directions), where both of the last terms in (10) contribute, namely if \vec{k}_1 and \vec{k}_2 are within the “recoil cone” of both detection directions \vec{k} and \vec{k}' . For identical \vec{k} and \vec{k}' this interference give precisely the factor 2 increase of coincidences compared to the case of random directions. We also note that the enhanced coincidences occur within a solid angle specified precisely by this “recoil cone”. The next section develops our model one step further and carries out the calculation for the special choice of a Gaussian Ansatz for the function $\psi(\vec{k}_1, \vec{k}_2)$.

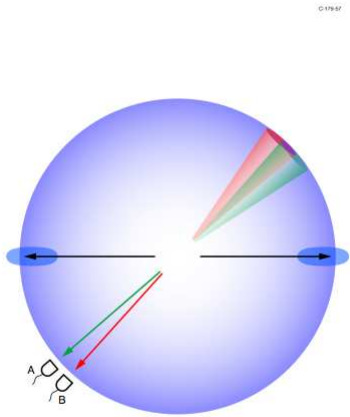


FIG. 3: State of four atoms, with two atoms detected in nearly parallel directions. The detected atoms are not unambiguously identified with their recoiling partners if their momentum distributions are wide enough to overlap. This leads to an interference term in the coincidence counting yield in detectors A and B as expressed by the formal expression (10).

V. RESULT FOR A SIMPLE ANSATZ FOR

$$\psi(\vec{k}_1, \vec{k}_2)$$

As stated above, the Bogoliubov approximation to the actual state can be found numerically by solution of linear wave equations. In this section, we shall rather take a simpler approach by making an Ansatz for the shape of the function ψ by appealing to the dynamics and the conservation laws valid in the bipartite collision dynamics.

Energy conservation, which is effectively enforced during the temporal solution of the Schrödinger equation, suggests that the atomic pair state $\psi(\vec{k}_1, \vec{k}_2)$ describes particles with the same energy as the incident condensate particles. Momentum conservation suggests that they also have the same total momentum as the colliding pair. Due to the finite size of the colliding clouds, this does not strictly imply that the two atoms must have exactly opposite momenta. The finite size of the collision zone implies a quantum mechanical momentum uncertainty, and if contributions from small regions (with correspondingly large total momentum uncertainty) are added coherently the resulting phase matching condition is not sharp if the entire collision zone has finite size. There are therefore quantum fluctuations of both the modulus and direction of the momenta. When the particles escape from the collision zone as illustrated in Fig. 1., they are also repelled by the mean field interaction with the two condensate

components. Here we will not try to describe accurately the effect of this interaction, which is in any case small when atoms leave a condensate after receiving an initial kinetic energy large compared to the chemical potential [22]. Note, however, that such mean field repulsion has in fact turned out to be tractable in the atom laser output from a condensate, where a generalized ABCD matrix formalism yields an analytical description of the propagation [23].

Define coordinates such that the nearly parallel \vec{k}, \vec{k}' of interest are close to the negative z -direction. Their partners at \vec{k}_1, \vec{k}_2 must both be close to the positive z -direction, and we shall assume the z -coordinates to be equal and opposite and only look at their x and y components. Their widths are related to the wave functions of the colliding condensates, both due to the amplitude of collisions out of these condensates and due to the mean field repulsion, and they are thus in general anisotropic. We restrict for simplicity the integration to one transverse coordinate (putting vector arrows on the arguments will yield the 2D result), and we assume that a single pair is described by a wave function, where the wave function amplitude for the recoiling partner has a bell shaped profile, that we for simplicity approximate by a Gaussian, centered at minus the coordinates of the detected particle. The width of this wave function is parametrized by a momentum width K which thus represents both the momentum width of the colliding condensate particles and the acceleration due to the mean field.

Assuming thus the last two terms in (10) to be of such Gaussian shape, and ignoring the first term which vanishes for the geometry studied, we can explicitly calculate the coincidence signal:

$$\begin{aligned} F(\vec{k}, \vec{k}') &\propto \int dk_1 dk_2 \\ &\quad |\exp(-((k_1 + k)^2 + (k_2 + k')^2)/2K^2) \\ &\quad + \exp(-((k_1 + k')^2 + (k_2 + k)^2)/2K^2)|^2 \\ &\propto 1 + \exp(-(k - k')^2/2K^2), \end{aligned} \quad (11)$$

where we recall that k and k' here refer to (small) transverse coordinates of the detector directions with respect to a given axis, i.e. $(k - k')$ is the radial momentum of the outgoing particles multiplied with their mutual (small) angle in radians.

We recover the Hanbury Brown and Twiss correlations, and we observe that the correlations persist for final state momenta within a distance from each other of the order of the quantum mechanical uncertainty of the total momentum of the atom pair escaping the collision zone. This is in accord with our interpretation in terms of the interference between the indistinguishable components illustrated as the overlapping recoil cones in Fig.1 c), that leads to the last term in (11) depending on both k and k' , whereas the direct terms lose the k and k' dependence due to the Gaussian integrals.

It is interesting to note, that Eq.(11) follows from a two-state amplitude $\psi(k, k') \propto \exp(-(k + k')^2/2K^2)$

for transverse momentum components of atoms propagating in nearly opposite directions, and therefore the two atom component of Eq.(6) predicts a correlation of atoms in opposite directions with the dependence $|\psi(k, k')|^2 \propto \exp(-(k + k')^2/K^2)$. The Hanbury Brown and Twiss bunching thus occurs within a Gaussian width that is $\sqrt{2}$ times larger than the range of correlation of recoiling atomic momenta. This prediction for the Gaussian wave functions has been verified by more detailed analysis of the full 3D propagation, [5]. One way to understand the broadening is to recognize that the density dependence of the pair production mechanism results in a source which is spatially narrower than the condensates themselves.

Although we have based our analysis on the four-atom component of the full many body states, we have argued that a calculation based on the full state would yield the same results, and in particular that the HBT correlation amounts to a factor of two in parallel directions while the correlation in opposite directions is not limited by this factor. When multiple scattering is neglected, the complete many-body problem is solved by the Bogoliubov-de Gennes equations, and the resulting Gaussian/thermal character of the many-body state is fully accounted for by the second moments. This does not imply, however, that one would get the same quantitative results for scattering of few and many atoms. If the normalized wave function for a single scattered pair in the case of a low scattering probability is denoted $\chi(\vec{k}_1, \vec{k}_2)$, it may be reasonable to describe the collision process by the effective Hamiltonian $H = \kappa \int d\vec{k}_1 d\vec{k}_2 \chi(\vec{k}_1, \vec{k}_2) \hat{\Psi}^\dagger(\vec{k}_1) \hat{\Psi}^\dagger(\vec{k}_2) + h.c.$, where κ is a coupling strength. The unitary time evolution operator is the exponential of this operator multiplied by $(t/i\hbar)$ or integrated over a suitable time interval. We note that this does not generally result in an expression for $\psi(\vec{k}_1, \vec{k}_2)$ in (4) which is proportional to $\chi(\vec{k}_1, \vec{k}_2)$. In the case of single mode squeezing, it is known that one must evaluate the hyperbolic tangent function of the squeezing parameter to convert the squeezing operator to the normal order form [11], and in our general multimode case, normal ordering is accomplished by evaluating the *tanh* function of a matrix argument [12, 13]. For small arguments, in the perturbative regime of spontaneous four wave mixing, *tanh* is a linear function, and we get the same momentum dependence. Outside the perturbative regime, we retain the factor 2 bunching effect by our general argument, but the precise shape of the correlation peak may be modified.

VI. DISCUSSION

We have presented a simple interpretation of the observed Hanbury Brown and Twiss correlations observed in the elastic scattering of Bose-Einstein condensates. We emphasize, that in order to make quantitative predictions, it is necessary to make a more elaborate calculation of the pair formation and the propagation of the atoms

both in free space and in the regions where the mean field of the condensate components act as a perturbing potential. Such a description is offered by the Bogoliubov theory in Refs.[8, 9, 10], and we note that [10] as well as [4] also provide numerical evidence for the density correlations discussed in the present paper. Our interpretation relies on the structural property of the solutions to the Bogoliubov theory (4), but it proceeds by applying a different physical reasoning which recognizes that the two detected particles are accompanied by collision partners propagating in the opposite directions, and we hence observe part of a four-atom state. This is an appealing picture, in particular because the prediction of the coincidence signal, and in particular its width, relates to the transverse spreading of the pair wave functions of oppositely propagating atoms after the bipartite collisions.

As we discussed in the text, when observed from only one direction, the reduced density matrix of the expanding atomic cloud is similar to a thermal state. This density matrix is sufficient to predict the outcome of any measurement on the observed part of the system, and it explains the experimental findings as an analogue of the observed bunching of the photons from a thermal/chaotic light source. The optical Hanbury Brown and Twiss experiment has a characteristic transverse spatial scale over which the correlation falls to unity, related to the transverse momentum distribution of the photons impinging on the detector, and in a similar manner we have a finite transverse coherence length in the atomic scattering experiment. We discussed the isotropic s-wave scattering, with possible corrections due to anisotropy of the colliding clouds and spatial phase matching. In addition, one may apply a periodic background potential, which may alter the energy dependence on the momentum vector of moving atoms, and hence modify the scattering profile [24, 25], and with confinement to one dimension, it may lead to highly selective population of specific momentum states with strong, observable quantum correlations [26, 27].

It is interesting to recall that a mixed quantum state, i.e., a density matrix, can always be formally obtained as the reduced state of a larger quantum system which is in a pure state, and in particular any thermal quantum state of a bosonic degree of freedom can be modelled by a pure squeezed state in a doubled tensor space. This is known as the "thermofields" formulation [28], and for example the single mode thermal state (2) can be obtained as the trace over one of the modes of a non-degenerate two mode squeezed state, as obtained, e.g., from a non-degenerate optical parametric oscillator (OPO),

$$|\psi_{OPO}\rangle \propto \sum_n t^n |n, n\rangle. \quad (12)$$

In our four-atom analysis, the apparently thermal state arriving at nearby detectors, is precisely part of such a larger system. The advantage of this insight is that the spatial scale of the extended state, in our case the probability distribution of the total momentum of scattered

atoms, directly yields the density correlations in the reduced density matrix.

Finally, if the collision occurs between two different bosonic species, the same kind of correlations will occur for the density correlations of each species, but not for the cross correlation, where the recoiling atoms are distinguishable, and hence do not interfere. For collisions between bosons and fermions the situation is different. Electrons have been demonstrated to show anti-bunching related to their fermionic character [29], and anti-bunching has also been demonstrated for neutral fermionic atoms [30, 31]. Collisions between a Bose condensate and a degenerate Fermi gas, where all fermions initially occupy orthogonal states, but where Pauli blocking forbids more than one atom ending up in the same

final state should lead to observable anti-bunching effects in the scattered bosons.

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